A simple six-line arrangement on a projective plane is obtained by a system of labelled six lines $L_1, L_2, \ldots, L_6$ with the conditions; (1) they are mutually different and (2) no three of them intersect at a point. We add the condition that (3) there is no conic tangent to all the lines. The main subject of this talk is to treat such arrangements on a projective plane over a finite prime field.

Before entering into the main subject, we now explain some results on the real case. There are four types of simple six-line arrangements on a real projective plane. Among the four types, one is characterized by the existence of a hexagon and one is characterized by the condition that the conic tangent to any five lines of the six lines does not intersect the remaining line. The totality of systems of labelled six lines with conditions (1), (2) admits the action of the sixth symmetric group by permutations among six lines. The advantage of the condition (3) is that the action of the sixth symmetric group on the totality of systems of labelled six lines with conditions (1), (2), (3) naturally extends to that of the Weyl group $W(E_6)$ of type $E_6$. It is shown in J. Sekiguchi and M. Yoshida $W(E_6)$-action on the configuration space of 6 points of the real projective plane, Kyushu J. Math., 51 (1997) that $W(E_6)$ acts transitively on the set of systems of labelled six lines fixed by a group isomorphic to a fifth symmetric group and that this is decomposed into four orbits by the sixth symmetric group action. These four $S_6$-orbits are in a one to one correspondence with the four types of simple-six line arrangements mentioned above.

The purpose of this talk is to study what happens when we replace a real projective plane by a projective plane over a finite prime field. Let $p$ be a prime number, $F_p$ the field consisting of $p$ points and $P^2(F_p)$ the projective plane over $F_p$. Let $L_1, L_2, \ldots, L_6$ be six lines on $P^2(F_p)$ with the conditions (1), (2), (3). Then we shall show the following theorems.

**Theorem 1.** If 5 is a quadratic residue mod $p$, there is a system of labelled six-line arrangement on $P^2(F_p)$ fixed by a fifth symmetric group.

It is easy to determine systems of labelled six lines fixed by a fifth symmetric group. They are related with the diagonal surface of Clebsch. In fact, if 5 is a quadratic residue mod $p$, the twenty-seven lines on it are defined over $F_p$ and a system of labelled six lines fixed by a fifth symmetric group is obtained by blowing down the diagonal surface.

**Theorem 2.** Now assume that there is $n \in F_p$ such that $n^2 \equiv 5 \pmod{p}$. Then there is a system of labelled six lines fixed by a fifth symmetric group such that the conic tangent to any five lines of the six lines does not intersect the remaining line if and only if $\pm 2n - 5$ is a non-quadratic residue mod $p$.

It is well-known that for a prime $p$, 5 is a quadratic residue mod $p$ if and only if $p = 10k + 1$ or $p = 10k - 1$ for a positive integer $k$. The following theorem was conjectured by the author and later proved by T. Ibukiyama.

**Theorem 3.** For a prime $p$ with $5 < p$, there is $n \in F_p$ such that $n^2 \equiv 5 \pmod{p}$ and there is no $m \in F_p$ such that $m^2 \equiv 2n - 5 \pmod{p}$ if and only if $p = 10k - 1$ for a positive integer $k.