Indivisibility of class numbers of imaginary quadratic function fields

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Abstract. We show that for an odd prime number $l$, there are infinitely many imaginary quadratic extensions $F$ over the rational function field $K = \mathbb{F}_q(T)$ such that the class number of $F$ is not divisible by $l$. This work is published in Acta Arithmetica 132.4 (2008).

Let $p$ be an odd prime number, $q$ a power of $p$ and $\mathbb{F}_q$ the finite field with cardinality $q$. Let $T$ be an indeterminate and $K = \mathbb{F}_q(T)$ the rational function field.

Let $l$ be an odd prime number. Friesen [3], Cardon and Murty [1], respectively, proved that there are infinitely many real and imaginary, respectively, quadratic extensions $F$ over $K$ such that the class number of $F$ is divisible by $l$.

In [6], Kimura proved that there are infinitely many quadratic extensions $F$ over $K$ such that the class number of $F$ is not divisible by 3. For an odd prime number $l$, Ichimura [5] constructed infinitely many imaginary quadratic extensions $F$ over $K$ such that the class number of $F$ is not divisible by $l$, when the order of $q$ mod $l$ in the multiplicative group $(\mathbb{Z}/l\mathbb{Z})^*$ is odd or $l = p$.

In this talk, we shall show the following theorem.

**Theorem 0.1** Let $l$ be an odd prime number. Then there are infinitely many imaginary quadratic extensions $F$ over $K$ such that the class number of $F$ is not divisible by $l$.

Theorem 0.1 is a function field analogue of Hartung’s work [4] on the imaginary quadratic number fields. To prove this theorem, following Hartung’s
idea in [4], we shall use the class number relation over function fields which
is developed by Yu [7].

References

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